Sequences

38. Prove that the sequence with general term $a_n = \frac{1}{\frac{n}{2n+1}}$ is:

(a) Monotonic,

(b) Bounded above,

(c) Bounded below,

(d) Bounded,

(e) Convergent.

39. A real sequence is given with the general term:

$$a_n = 2 + \frac{1}{\frac{n}{n+1}}.$$

Write the first, fifth, tenth, hundredth, and thousandth terms of the sequence. Justify whether the sequence is:

(a) Monotonic,

(b) Bounded above,

(c) Bounded below,

(d) Bounded.

If the sequence is convergent, find its limit.

40. A real sequence is defined by the first term $x_1 = 1$ and the recursive relation:

$$x_{n+1} = \frac{2x_n + 3}{x_n + 2}.$$

Justify whether the sequence is:

(a) Monotonic,

(b) Bounded above,

(c) Bounded below,

(d) Bounded.

If the sequence is convergent, find its limit.

41. Calculate the limit:

$$\lim_{n\to\infty} (\sqrt{n-3} - \sqrt{n+3}).$$

42. A real sequence $(a_n)_{n\in\mathbb{N}}$ is given by the general term:

$$a_n = \frac{n}{\sqrt{2+3n^2}}.$$

(a) Show that the sequence $(a_n)_{n\in\mathbb{N}}$ is monotonic.

(b) Show that the sequence $(a_n)_{n\in\mathbb{N}}$ is bounded.

(c) Find the limit of the sequence $(a_n)_{n\in\mathbb{N}}$.

43. A real sequence $(a_n)_{n\in\mathbb{N}}$ is defined recursively by:

$$a_{n+1} = 5\sqrt{a_n - 1} - 3,$$

with the first term $a_1 = 5$.

(a) Show that the sequence $(a_n)_{n\in\mathbb{N}}$ is monotonic.

(b) Show that the sequence $(a_n)_{n\in\mathbb{N}}$ is bounded.

(c) Find the limit of the sequence $(a_n)_{n\in\mathbb{N}}$.

All above math problems are taken from the following website:
`https://osebje.famnit.upr.si/~penjic/teaching.html`.
The reader can find all solutions to the given problems on the same page.